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The Human Heart Laid Bare

By Michelle Sipics

A professor stands at a blackboard, chalk in hand. He's explaining some mathematical concept, hands flying as he scribbles equation after equation, ending with the inevitable "any questions?"

Most of the students sit quietly, but one is openly puzzled. He stares at the board, trying to follow the equations through their logical progression, but sighs. "I just don't see it," he tells his professor.

If the concepts being explained by the hypothetical professor were related to the operation of the human heart, Elizabeth Cherry and Flavio Fenton of Cornell's College of Veterinary Medicine could provide some help. The two researchers received an honorable mention in the 2006 Science and Engineering Visualization Challenge for their interactive program "Cardiac Bioelectricity and Arrhythmias." The journal *Science* and the National Science Foundation established the Challenge in 2003 to "encourage recognition of the visual and conceptual beauty of science and engineering."

Cherry and Fenton's winning program allows users to view animations of a heart beating irregularly—in this case, because of an arrhythmia—and offers educational information on normal and abnormal heart operation as well as simulations of various forms of cardiac activity. Users can view animations of a heart beating normally, or choose to watch a heart with a particular form of arrhythmia; rotate, zoom, or pan a beating heart while it is in motion in order to view it from different perspectives; and even watch a simulated fibrillation, where the still-active animation can be rotated on any of the three axes, among other features.

Cherry, who has an undergraduate degree in mathematics and a doctorate in computer science, became interested in studying cardiac activity while in graduate school.

"I was drawn to the beauty and complexity of the patterns produced by the propagating waves," she says. She also points to the interdisciplinary nature of the topic as a motivational factor: "[It] gave me an opportunity to study not only numerical methods

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"Still Life," created by Luc Bénard using visualizations from Richard Palais' 3D-XplorMath program, was awarded first place in the illustration category of the Science/NSF 2006 Visualization Challenge. (See article on page 4.) The surfaces in the image, clockwise from lower left, are: Klein Bottle, Symmetric 4-Noid, Breather, Bryant–Kusner Boy's Surface, and the Sievert–Enneper Surface. Readers are encouraged to visit http://www.math.uci.edu/collection.jpg, where a high-resolution, color version of the image can be found. Courtesy of Richard Palais, University of California, Irvine, and Luc Bénard.

Show, Don't

In nine weeks on the science desk of an urban newspaper, the 2006 SIAM/AAAS media fellow improved her writing skills, learned new science, and gained new respect for science journalism.

By Katharine Ott

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for solving partial differential equations, but also the underlying physiology," she explains.

Fenton, who holds a doctorate in physics, also traces his interest to graduate school. In his case, however, the path began in complex systems and a project on reaction–diffusion equations, which eventually led to models of cardiac electrical dynamics and the idea of modeling arrhythmias.

"At the time I was in graduate school, the first simulations of spiral waves in cardiac tissue were being performed in 2D," he says. While normal electrical impulses in cardiac tissue are generally smooth, spiral waves can sometimes develop; if they subsequently become unstable and break up, the resulting impulses can cause dangerous, even fatal arrhythmias. Despite the work being done in two dimensions at that time, Fenton says, "no one knew what effect the fiber rotation that occurs naturally in 3D tissue and that produces rotational anisotropy had on spiral wave stability." That uncertainty left an intriguing open problem for Fenton to study.

"[It] was attractive because it required learning a lot of different skills," he says. "Fast computational methods, biology and physiology, as well as programming and visualization in 3D."

Applying those skills, Cherry and Fenton eventually came up with the cardiac bioelectricity and arrhythmias program, which is freely available on their Web site (see endnote for more information). Multiple mathematical concepts and a range of geometries were used in the development of the program and its integrated animations, Cherry explains.

"The models of electrical activity stem from Hodgkin–Huxley-style models of excitable physiological systems," she says. Essentially, each cell is described by a system of nonlinear ordinary differential equations. Continuum approximation and diffusive coupling methods are used to connect the cells; the result is a system of nonlinear partial differential equations. Those equations are then discretized in space and time using Taylor series methods.

The modeling presents significant computational challenges, says Cherry, "not only because of the large number of variables and parameters that must be tracked at each position in space, but because of the variety of space and time scales present" in the heart—space scales in the researchers' simulations range over three orders of magnitude, and time scales over six. The complexity of the problem, Fenton adds, increases with the incorporation of further realistic elements, such as geometric structures with irregular boundaries, or anatomical variations in physiological properties.

The researchers met those challenges, however, and the judges of the Visualization Challenge cited their program's use of "a combination of words, pictures, computer simulations and animations" to provide an interactive experience that can educate users about arrhythmias.

For their part, Cherry and Fenton were extremely gratified to learn that their program, which they say is in many ways the culmination of a decade of research, is providing others with a way to learn about cardiac rhythms.

"It was a wonderful opportunity to synthesize sophisticated visualization modalities to educate others," Cherry says, and allow them "to experience the same complex dynamics and patterns that drew us into the field and keep us there today."

Their plans for the research don't stop there. Eventually, the duo would like to have a realistic, accurate, and reliable model of the human heart that includes electrical, intracellular calcium, and contraction dynamics—one that can be used for studying all aspects of arrhythmias.

"Cardiac dynamics is a very rich system to study, complete with bifurcations, nonlinear wave propagation, and complex patterns," says Fenton. "Hopefully in the future people will consider cardiac dynamics on par with neurobiology in terms of introducing students from mathematics, physics, biomedical engineering, and biology to excitable physiological systems."

Cherry echoes his optimism. "Cardiac bioelectricity has a richness of dynamics and a diversity of behavior," she says. "We hope in the future to produce additional educational materials in concert with our basic research, to draw others into the field."

Interested readers can find Cherry and Fenton's program at http://www.vet.cornell.edu/news/FentonCherry/Media/main.html.

Fenton's tutorial "Excitable Media" (Java Applets) is part of an extensive set of tutorials available on DSWeb, the Web site of the SIAM Activity Group on Dynamical Systems.

Details about the Science*/NSF Visualization Challenge are available at http://www.sciencemag.org/sciext/vis2006/.*

Michelle Sipics is a contributing editor at SIAM News.

Visualizing Mathe

Richard Palais, a mathematician at the University of California, Irvine, dates his work in mathematical visualization to the early 1990s, when his wife, Chuu-lian Terng, was researching the relationship between soliton mathematics and submanifold geometry. Terng wanted to visualize certain surfaces in order to better understand the relationship; Palais was able to convert mathematical ideas into numerical algorithms and, eventually, to bring visual representations of the surfaces into being. That experience provided the inspiration for 3D-XplorMath, a program for the Macintosh platform that Palais developed and has been updating ever since.

Palais has made 3D-XplorMath freely available since the late 1990s. One day early this year, he recalls, a message arrived "out of the blue" from one user, a graphic artist named Luc Bénard. Attached to Bénard's message was a digital image whose contents looked extremely familiar to Palais. Bénard had taken visualizations created by 3D-XplorMath, imported them into the 3D rendering software Bryce, and transformed what Palais recognized as "his children" into "glass sculpture"—a transformation that, he says, "seemed like pure magic."

The judges of the *Science*/NSF Visualization Challenge, seeming to agree, awarded Palais and Bénard first place in the illustration category for their image "Still Life."

The image, which contains visualizations of